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An Adaptive Weighted Differential Game Guidance Law

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Abstract

For intercepting modern high maneuverable targets, a novel adaptive weighted differential game guidance law based on the game theory of mixed strategy is proposed, combining two guidance laws which are derived from the perfect and imperfect information pattern, respectively. The weights vary according to the estimated error of the target's acceleration, the guidance law is generated by directly using the estimation of target's acceleration when the estimated error is small, and a differential game guidance law with adaptive penalty coefficient is implemented when the estimated error is large. The adaptive penalty coefficients are not constants and they can be adjusted with current target maneuverability. The superior homing performance of the new guidance law is verified by computer simulations.

Keywords: differential games; guidance laws; adaptive weight; penalty coefficient; information patterns; game theory

1. Introduction

Modern aerial targets, such as unmanned aerial vehicle (UAV), tactical ballistic missile (TBM) and cruise missile (CM), are of high maneuverability. The successful interception of these targets, much less vulnerable than an aircraft, requires the interceptor missile to have a superior guidance law. Currently used guidance laws are developed based on a linear quadratic optimal control formulation. Since the target maneuvers are independently controlled, formulating the interception as an optimal control problem is not an adequate approach [1]. The mathematical framework for analyzing conflicts between two independent agents is in the realm of dynamic games. Thus, the scenario of intercepting a maneuverable target can be formulated as a zero-sum pursuit-evasion differential game [2-3].

Based on some simplified assumptions, the original pursuit-evasion problem can be transformed into a linear differential game. Anderson [4] clearly stated the superiority of differential game guidance laws over optimal control guidance laws, because differential game guidance law is less sensitive to errors in the estimation of current target acceleration. Anderson assumed a perfect target airframe/autopilot response and considered both perfect and first order missile responses. Then, Shinar [5-6] and Chen [7], et al. extended the results of a target with first order response dynamics. Shima and Shinar [8] modified the guidance law, based on the assumption that both players have variable speed and bounded acceleration. To implement in a noise corrupted environment, Shima and Weiss [9] took into account the inherent delay in estimating the target's acceleration, and improved the zero-effort miss (ZEM) accuracy by computing the center of the target acceleration reachable set. Oshman and Arad [10] used imagery data to decrease the target maneuver reachable set, and the miss distance (the distance of the closed approach, or the smallest norm of the separation vector) was significantly decreased via the simulation. Turetsky [11-12], based on a linear-quadratic differential game formulation, indicated that the interceptor can guaran-

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tee an arbitrarily small miss distance without exceeding the control constraints if the penalty coefficients were chosen properly. It has been reported that the cost function of such zero-sum differential game is the miss distance, and the derived guidance law is a kind of bang-bang control about ZEM [5-10]. The cost function is the weighted sum of three quadratic forms: the square of the miss distance and two penalty terms, which are the integrals of the respective control energy of the players [4, 11-12]. A critical caveat of this method is that the coefficients of the respective control energy are assumed to be constants. This could yield disappointing results when the target implements maneuvers with various magnitudes, as the guidance laws with constant parameters would not be able to guarantee an adequate homing accuracy [13].

Realistic interception is characterized by an imperfect and asymmetrical information pattern. The evading target usually maneuvers randomly and has no information on the relative state of the interceptor; meanwhile the interceptor has noise-corrupted measurements on the relative position of its target. In this study, we use the adaptive filter associated with 'current' statistical model for estimation. We aim to take advantage of the target's estimation, and adjust the penalty coefficients according to the estimation; the novel guidance law is formulated based on current information pattern.

2. Game Formulation

A two-dimensional interception scenario between interceptor missile and its target is formulated. Figure 1 shows a schematic view of this planar engagement geometry. The x-axis of the coordinate system is aligned with the initial line of sight (LOS) [14]; R is the target-missile relative range; V_M and V_T are the velocities of missile and target, respectively; q is the LOS angle; and θ_M and θ_T are the flight path angles of missile and target, respectively.

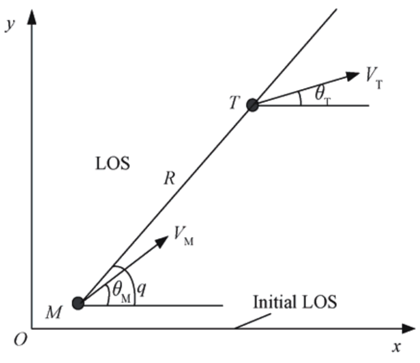

Fig. 1 Target-missile engagement in plane.

The engagement geometry can be described as follows:

$$\begin{cases} R\dot{q} = v_M \sin(q-\theta_M) - v_T \sin(q-\theta_T) \\ \dot{R} = v_T \cos(q-\theta_T) - v_M \cos(q-\theta_M) \end{cases} \tag{1}$$

where $\dot{R}$ is the target-missile relative velocity, and $\dot{q}$ the LOS angular rate.

Note that the angles are sufficiently small ($q-\theta_M=0$, $q-\theta_T=0$). Assume VT and VM are constant velocities [15]. By denoting $x=q$, the corresponding equation becomes

$$\dot{x} = -\frac{2\dot{R}}{R}x - \frac{1}{R}u + \frac{1}{R}v \tag{2}$$

where u and v represent the control variable of missile and its target, respectively.

A linear-quadratic differential game is formulated for system (2) with the performance index

$$J = \frac{a}{2}x^2(t_f) + \frac{1}{2}\int_{t_0}^{t_f}(bu^2 - cv^2)\,d\tau \tag{3}$$

where $a \geq 0$, $b > 0$ and $c > 0$ are the penalty coefficients, selected by the guidance analyst; t_0 and t_f are the initial and final time of the game, respectively. It is well known that zeroing the LOS angular rate can guarantee an adequate homing accuracy. Hence, a usually tends to positive infinity. The pursuer (missile) wishes to minimize the performance index (Eq. (3)), while the evader (target) aims to maximize it both using the feedback strategies u and v, respectively.

3. Game Solution with Different Information Patterns

Based on the cost function (Eq. (3)) and the dynamics (Eq. (2)), the Hamiltonian of the problem is

$$H = \frac{1}{2}(bu^2 - cv^2) + \lambda\left(-\frac{2\dot{R}}{R}x - \frac{1}{R}u + \frac{1}{R}v\right) \tag{4}$$

where λ is the co-state variable, and it is satisfied the following equation and terminal condition

$$\dot{\lambda} = -\frac{\partial H}{\partial x} = \lambda\frac{2\dot{R}}{R},\quad \lambda(t_f) = ax(t_f) \tag{5}$$

The solution of co-state variable Eq. (5) is

$$\lambda = a e^{-2\dot{R}(t_f-t)/R} x(t_f) \tag{6}$$

By the necessary conditions

$$\frac{\partial H}{\partial u} = 0,\ \frac{\partial H}{\partial v} = 0$$

the optimal strategies are obtained as follows:

$$u = \frac{\lambda}{bR} = \frac{a}{bR}e^{-2\dot{R}(t_f-t)/R}\,x(t_f) \tag{7}$$

$$v = \frac{\lambda}{cR} = \frac{a}{cR}e^{-2\dot{R}(t_f-t)/R}\,x(t_f) \tag{8}$$

From Eqs. (7)-(8), the optimal strategies are determined by the final state $x(t_f)$, and different final states can be computed from different information patterns.

3.1. Feedback strategies with perfect information

"Perfect information" is defined by the scenario where each player has perfect knowledge of the prior information of his opponent [16]. Based on this assumption, the target implements the optimal strategy as Eq. (8). Substituting strategies (Eqs. (7)-(8)) into Eq. (2), and integrating Eq. (2) from any time t to t_f lead to the following expression of final state $x(t_f)$:

$$\begin{aligned}x(t_f) &= \Phi(t_f,t)x(t)+\int_t^{t_f}\Phi(t_f,s)\left(-\frac{1}{R}u+\frac{1}{R}v\right)ds = \\ &\mathrm{e}^{-2\chi}x(t)+\int_t^{t_f}\mathrm{e}^{-2\chi}\left(-\frac{1}{R}u+\frac{1}{R}v\right)ds = \\ &\mathrm{e}^{-2\chi}x(t)+\int_t^{t_f}\mathrm{e}^{-2\chi}\left(-\frac{1}{R}\cdot\frac{a}{bR}+\frac{1}{R}\cdot\frac{a}{cR}\right)\mathrm{e}^{-2\chi}x(t_f)ds = \\ &\mathrm{e}^{-2\chi}x(t)-\frac{a}{R}x(t_f)\left(\frac{1}{4b\dot{R}}-\frac{1}{4c\dot{R}}\right)(1-\mathrm{e}^{-4\chi})\end{aligned} \tag{9}$$

where $\Phi(\cdot)$ is the transition coefficient, $\chi=\dot{R}(t_f-t)/R$, $x(t_f)$ is given by

$$x(t_f)=\frac{\mathrm{e}^{-2\chi}}{1+\frac{a}{R}\left(\frac{1}{4b\dot{R}}-\frac{1}{4c\dot{R}}\right)(1-\mathrm{e}^{-4\chi})}x(t) \tag{10}$$

Substituting $x(t_f)$ into Eqs. (7)-(8), yields the following optimal feedback strategies:

$$u=\frac{a}{bR}\cdot\frac{\mathrm{e}^{-4\chi}}{1+\frac{a}{R}\left(\frac{1}{4b\dot{R}}-\frac{1}{4c\dot{R}}\right)(1-\mathrm{e}^{-4\chi})}x(t) \tag{11}$$

$$v=\frac{a}{cR}\cdot\frac{\mathrm{e}^{-4\chi}}{1+\frac{a}{R}\left(\frac{1}{4b\dot{R}}-\frac{1}{4c\dot{R}}\right)(1-\mathrm{e}^{-4\chi})}x(t) \tag{12}$$

Note that the remaining time $t_f-t\approx R/\left|\dot{R}\right|=-R/\dot{R}$, i.e., $\chi=-1$. Thus, the interceptor's optimal strategy u is

$$u=\frac{a}{bR}\cdot\frac{\mathrm{e}^{4}}{1+\frac{a}{R}\left(\frac{1}{4b\dot{R}}-\frac{1}{4c\dot{R}}\right)(1-\mathrm{e}^{4})}x(t) \tag{13}$$

3.2. Feedback strategies with imperfect information

In an imperfect information interception scenario, the evading target usually has no information about the relative state of the interceptor and maneuvers randomly.

1) Case 1: Target implements no maneuver, $v=0$.

This is equivalent to the limit case $c\to+\infty$. From Eq. (13), the interceptor's optimal strategy u is given by

$$u=\frac{a\mathrm{e}^4}{bR\left[1+\frac{a}{R}\cdot\frac{1}{4b\dot{R}}(1-e^4)\right]}x(t) \tag{14}$$

Set a to be positive infinity in Eq. (14). The optimal strategy u can be written as

$$u=\frac{\mathrm{e}^4}{\frac{1}{4\dot{R}}(1-\mathrm{e}^4)}x(t)=\frac{4\mathrm{e}^4}{1-\mathrm{e}^4}\cdot$$

$$\dot{R}x(t)\approx-4\dot{R}x(t) \tag{15}$$

The optimal strategy expressed by Eq. (15) has also been derived by a previous report (Ref. [15]), where assuming $b=-1/\dot{R}$, the optimal strategy gain calculated based on that report is 3, whereas our equation here calculates it to be 4 with no assumptions about b.

2) Case 2: Target's strategy is a known value v ($\neq 0$).

In this case, substitute target's strategy v into Eq. (2), and integrate Eq. (2) from any time t to t_f. The final time state $x(t_f)$ is therefore

$$\begin{aligned}x(t_f) &= \Phi(t_f,t)x(t)+\int_t^{t_f}\Phi(t_f,s)\left(-\frac{1}{R}u+\frac{1}{R}v\right)ds = \\ &\mathrm{e}^{-2\chi}x(t)+\int_t^{t_f}\mathrm{e}^{-2\chi}\left(-\frac{1}{R}u+\frac{1}{R}v\right)ds = \\ &\mathrm{e}^{-2\chi}x(t)+\int_t^{t_f}\mathrm{e}^{-2\chi}\left(-\frac{1}{R}\cdot\frac{a}{bR}\mathrm{e}^{-2\chi}x(t_f)+\frac{1}{R}v\right)ds = \\ &\mathrm{e}^{-2\chi}x(t)+\frac{v}{R}\int_t^{t_f}\mathrm{e}^{-2\chi}d\tau-\frac{a}{bR^2}x(t_f)\int_t^{t_f}\mathrm{e}^{-4\chi}ds = \\ &\mathrm{e}^{-2\chi}x(t)+\frac{v}{2\dot{R}}(1-\mathrm{e}^{-2\chi})-\frac{a}{4bR\dot{R}}(1-\mathrm{e}^{-4\chi})x(t_f)\end{aligned} \tag{16}$$

Thus, $x(t_f)$ is given by

$$x(t_f)=\frac{\mathrm{e}^{-2\chi}x(t)+\frac{v}{2\dot{R}}(1-\mathrm{e}^{-2\chi})}{1+\frac{a}{4bR\dot{R}}(1-\mathrm{e}^{-4\chi})} \tag{17}$$

Assuming $\chi=-1$ and substituting Eq. (17) into Eq. (7) yields the feedback optimal strategy u:

$$\begin{aligned}u &= \frac{a}{bR}\mathrm{e}^2\frac{\mathrm{e}^2x(t)+\frac{v}{2\dot{R}}(1-\mathrm{e}^2)}{1+\frac{a}{4bR\dot{R}}(1-\mathrm{e}^4)} = \\ &\frac{a\mathrm{e}^4x(t)+\frac{av}{2\dot{R}}\mathrm{e}^2(1-\mathrm{e}^2)}{bR+\frac{a}{4\dot{R}}(1-\mathrm{e}^4)}=\frac{4a\dot{R}\mathrm{e}^4x(t)+2a\mathrm{e}^2(1-\mathrm{e}^2)v}{4\dot{R}bR+a(1-\mathrm{e}^4)}\end{aligned} \tag{18}$$

Set a to be positive infinity in Eq. (18) and the optimal strategy u can be written as

$$\begin{aligned}u &= \frac{4\mathrm{e}^4\dot{R}x(t)+2\mathrm{e}^2(1-\mathrm{e}^2)v}{1-e^4}=4\dot{R}\frac{\mathrm{e}^4}{1-\mathrm{e}^4}x(t)+ \\ &2\mathrm{e}^2\frac{1-\mathrm{e}^2}{1-\mathrm{e}^4}v\approx-4\dot{R}x(t)+1.76v\end{aligned} \tag{19}$$

In the perfect information pattern, penalty coefficients b and c should be chosen properly according to target's maneuver acceleration. When evader takes high maneuver, penalty coefficient c should be close to b. Otherwise, c should be larger than b. In the imperfect information pattern, the second term of Eq. (19) is the target strategy (maneuver acceleration). So, target acceleration should be estimated in both cases above.

4. Adaptive Estimation of Target Acceleration Based on 'Current' Statistical Model

4.1. 'Current' statistical model

There is a physical relationship between the state variable (acceleration) and the mean value of the state noise [17]. In most cases, the random disturbance of acceleration is not driven by white noise, but by a time-correlated non-zero means value. A target is maneuvering with certain acceleration at present, and the calculated region of acceleration, which can be taken in the next instant, is limited, being around the 'current' acceleration.

The 'current' statistical model of discrete-time state estimation is given as

$$\boldsymbol{X}(k+1)=\boldsymbol{\varphi}(k+1|k)\boldsymbol{X}(k)+\boldsymbol{U}(k)\bar{a}_{\mathrm{T}}(k)+\boldsymbol{W}(k) \tag{20}$$

where $\boldsymbol{X}=[x_{\mathrm{T}} \quad v_{\mathrm{T}} \quad a_{\mathrm{T}}]^{\mathrm{T}}$ denotes state vector; x_{T}, v_{T} and a_{T} are the target's position, velocity, and acceleration, respectively; $\bar{a}_{\mathrm{T}}(k)$ is the mean value of current maneuver acceleration; transition matrix $\boldsymbol{\varphi}(k+1|k)$ is

$$\boldsymbol{\varphi}(k+1|k)=\begin{bmatrix} 1 & T & (-1+\tau T+\mathrm{e}^{-\tau T})/\tau^2 \\ 0 & 1 & (1-\mathrm{e}^{-\tau T})/\tau \\ 0 & 0 & \mathrm{e}^{-\tau T} \end{bmatrix} \tag{21}$$

and

$$\boldsymbol{U}(k)=\begin{bmatrix} -T/\tau+T^2/2+(1-\mathrm{e}^{-\tau T})/\tau^2 \\ T-(1-\mathrm{e}^{-\tau T})/\tau \\ 1-\mathrm{e}^{-\tau T} \end{bmatrix} \tag{22}$$

where T is the sampling interval, τ the reciprocal of maneuvering time constant.

$\boldsymbol{W}(k)$ is the zero-mean discrete white noise sequence with covariance $\boldsymbol{S}=2\tau\sigma_{\mathrm{a}}^2\boldsymbol{Q}$, σ_{a}^2 the variance of target acceleration, and $\boldsymbol{Q}$ the constant matrix concerned with τ and T [18].

$$\boldsymbol{Q}=\begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{12} & q_{22} & q_{23} \\ q_{13} & q_{23} & q_{33} \end{bmatrix} \tag{23}$$

where

$$q_{11}=\frac{1}{2\tau^5}\left(1-\mathrm{e}^{-2\tau T}+2\tau T+\frac{2\tau^3T^3}{3}-4\tau T\mathrm{e}^{-\tau T}-2\tau^2T^2\right)$$

$$q_{12}=\frac{1}{2\tau^4}(\mathrm{e}^{-2\tau T}+1-2\mathrm{e}^{-\tau T}+2\tau T\mathrm{e}^{-\tau T}-2\tau T+2\tau^2T^2)$$

$$q_{13}=\frac{1}{2\tau^3}(1-\mathrm{e}^{-2\tau T}-2\tau T\mathrm{e}^{-\tau T})$$

$$q_{22}=\frac{1}{2\tau^3}(4\mathrm{e}^{-\tau T}-3-\mathrm{e}^{-2\tau T}+2\tau T)$$

$$q_{23}=\frac{1}{2\tau^2}(1+\mathrm{e}^{-2\tau T}-2\mathrm{e}^{-\tau T})$$

$$q_{33}=\frac{1}{2\tau}(1-\mathrm{e}^{-2\tau T})$$

If only the noisy position data of target is available, the observation equation is

$$\boldsymbol{Y}(k)=\boldsymbol{H}(k)\boldsymbol{X}(k)+\boldsymbol{V}(k) \tag{24}$$

where $\boldsymbol{H}=[1 \quad 0 \quad 0]$, and $\boldsymbol{V}(k)$ is Gaussian noise with zero-mean and variance $\boldsymbol{G}(k)$.

4.2. Adaptive filter (AF) algorithm

The AF algorithm makes use of the standard Kalman filtering theory, and takes the one-step estimation of acceleration $\hat{a}_{\mathrm{T}}(k|k-1)$ as the current acceleration, namely the mean of stochastic maneuvering acceleration $\bar{a}_{\mathrm{T}}(k)=\hat{a}_{\mathrm{T}}(k|k-1)$. A variable target acceleration variance is given as follows.

When the current acceleration $\bar{a}_{\mathrm{T}}(k)\geq 0$, the target acceleration variance is taken as

$$\sigma_{\mathrm{a}}^2=\frac{4-\pi}{\pi}(a_{\max}-\hat{a}_{\mathrm{T}}(k-1|k-1))^2 \tag{25}$$

where $a_{\max}>0$ is the known positive acceleration limit of the target.

When the current acceleration $\bar{a}_{\mathrm{T}}(k)<0$, the target acceleration variance is taken as

$$\sigma_{\mathrm{a}}^2=\frac{4-\pi}{\pi}(a_{-\max}-\hat{a}_{\mathrm{T}}(k-1|k-1))^2 \tag{26}$$

where $a_{-\max}<0$ is the known negative acceleration limit of the target and may not have the same absolute value as $a_{\max}$.

As stated previously, an acceleration variance adaptive algorithm can be accomplished with the following steps.

1) The one-step-ahead prediction is

$$\hat{\boldsymbol{X}}(k|k-1)=\boldsymbol{\varphi}(k|k-1)\hat{\boldsymbol{X}}(k-1|k-1)+\boldsymbol{U}(k)\hat{a}_{\mathrm{T}}(k|k-1) \tag{27}$$

$$\boldsymbol{P}(k|k-1)=\boldsymbol{\varphi}(k|k-1)\boldsymbol{P}(k-1|k-1)\cdot\boldsymbol{\varphi}^{\mathrm{T}}(k|k-1)+\boldsymbol{S}(k) \tag{28}$$

2) The filter plus matrix is

$$\boldsymbol{K}(k)=\boldsymbol{P}(k|k-1)\boldsymbol{H}^{\mathrm{T}}(k)(\boldsymbol{H}(k)\cdot\boldsymbol{P}(k|k-1)\boldsymbol{H}^{\mathrm{T}}(k)+\boldsymbol{G}(k))^{-1} \tag{29}$$

3) The update of state and error matrix is

$$\hat{\boldsymbol{X}}(k|k)=\hat{\boldsymbol{X}}(k|k-1)+\boldsymbol{K}(k)(\boldsymbol{Y}(k)-\boldsymbol{H}(k)\hat{\boldsymbol{X}}(k|k-1)) \quad (30)$$

$$\boldsymbol{P}(k|k)=(\boldsymbol{I}-\boldsymbol{K}(k)\boldsymbol{H}(k))\boldsymbol{P}(k|k-1) \quad (31)$$

where $\boldsymbol{I}$ is the identity matrix.

4.3. Estimation result

In this simulation, time step T=0.01 s; the reciprocal of maneuvering time constant τ=0.01, $a_{\max}$=120 m/s^2, $a_{-\max}$=−120 m/s^2, and the observation variance G=400 m^2. Assume the target implements the following maneuver:

$$a_{\mathrm{T}}=\begin{cases}0 & t>10\\ 12g\sin(0.6t) & \text{Otherwise}\end{cases}$$

where g is the acceleration of gravity.

Figure 2 shows the estimation of the target's acceleration using the discrete AF based on 'current' statistical model. Estimated error of the acceleration is given in Fig. 3.

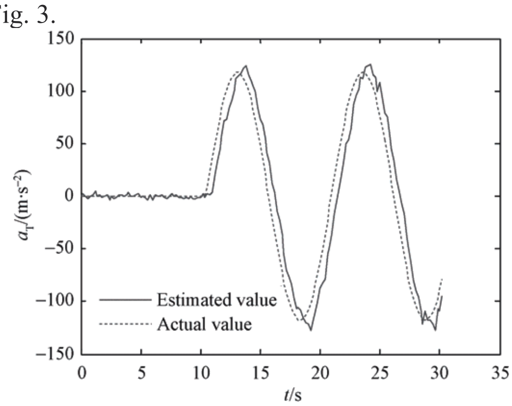

Fig. 2 Comparison of actual value and estimated value.

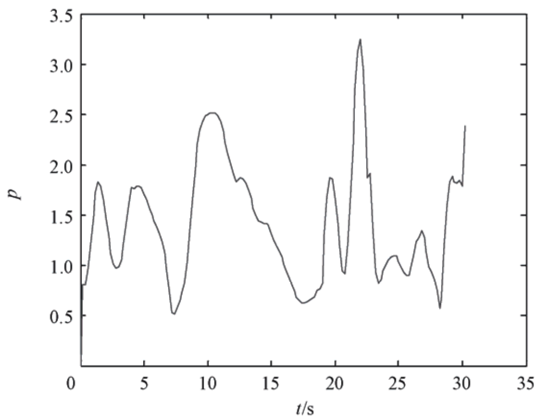

Fig. 3 Estimated error by AF algorithm.

In Fig. (3), p=P_{33} is the estimated error of acceleration P_{33} is the (3,3)th entry of $\boldsymbol{P}$, and $\boldsymbol{P}$ the covariance of error matrix in Eq. (31). Notice that the estimation delays when the target performs abrupt changes in its maneuver. As previously reported [19], the inherent delay in estimation results from the time used to identify a maneuver. Here, the time delay Δt is about 2 s.

5. The New Guidance Law

Estimation of target's acceleration is provided in Section 4. When the estimated error is large, the estimation with errors cannot be substituted into Eq. (19) to form the guidance law; otherwise, it will yield bad results. However, we can take advantage of the differential game's decreased sensitivity to estimated errors. First, based on the estimation results, the adaptive penalty coefficients of the cost function is designed properly, and then the two guidance laws, derived from different information patterns expressed by Eq. (13) and Eq. (18) are combined according to the estimated error.

5.1. Design of penalty coefficients

Previously the relationship between penalty coefficient b and c has been defined [11]:

$$c=\frac{b}{\mu^2\varepsilon^2} \quad (32)$$

where $\mu= a_{\mathrm{T}}^{\max}/a_{\mathrm{M}}^{\max}$ is the target/missile maneuverability ratio. Taking account of first order response dynamics, $\varepsilon=\tau_{\mathrm{T}}/\tau_{\mathrm{M}}$ is the ratio of the target/missile time constants [16]. c is therefore a constant.

The deficiency in the above design is that when a target of large maneuverability performs lower maneuvers, the penalty coefficient c is still small, therefore constant coefficient is unreasonable. However, we introduce two physical parameters: the current target/missile maneuverability ratio μ_{c} and the current relative estimated error δ_{c}:

$$\mu_{\mathrm{c}}=\frac{|\hat{a}_{\mathrm{T}}|}{a_{\mathrm{M}}^{\max}} \quad (33)$$

$$\delta_{\mathrm{c}}=\frac{p}{|\hat{a}_{\mathrm{T}}|} \quad (34)$$

where $\mu_{\mathrm{c}}\le\mu$.

The penalty coefficient c is defined as follows:

$$c=\frac{b\mathrm{e}^{-\delta_{\mathrm{c}}}}{\mu_{\mathrm{c}}^2\varepsilon^2} \quad (35)$$

If the estimated error δ_{c}=0, and $\hat{a}_{\mathrm{T}}=a_{\mathrm{M}}^{\max}$, then Eq. (35) equals Eq. (32). If $\hat{a}_{\mathrm{T}}$=0 (target performs no maneuver) then μ_{c}=0, resulting in $c\to+\infty$. This was illustrated by Case 1 in Section 3.2.

5.2. Adaptive weighted guidance law

Based on the game theory of mixed strategy [20], the new guidance law makes use of the estimation. When the estimated error is small, guidance law is generated by using the estimation of the target's acceleration directly. When the estimated error is large, a differential game guidance law with adaptive penalty coefficients is implemented. The weights W_1 and W_2 vary in accordance with the estimated error p:

$$\begin{cases} W_1 = \mathrm{e}^{-\sqrt{p}} \\ W_2 = 1 - \mathrm{e}^{-\sqrt{p}} \end{cases} \tag{36}$$

where $W_1+W_2=1$. A block diagram for this representation is shown in Fig. 4.

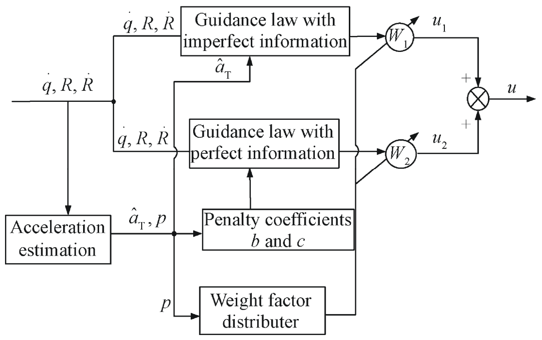

Fig. 4　Adaptive weight guidance law block diagram.

6. Numerical Simulation

Modern aerial targets such as UAV, TBM and CM have high maneuverability, with the assumption that their maximum magnitude of maneuverability is 12g, In this simulation, an interceptor missile is launched against a cruising UAV. At first, the UAV has no information about the in-coming interceptor. Three seconds later, the UAV becomes aware of the threat and initiates a bang-bang maneuver. Its strategy is expressed as

$$a_T = \begin{cases} 0 & t < 3 \\ 12g & 3 \le t < 7 \\ -12g & t \ge 7 \end{cases}$$

The time step T is 0.01 s. The residual time can be calculated by $t_r = R/\dot{R}$, and the remaining simulation parameters are listed in Table 1, where R_0 is the initial relative range between the target and the missile.

Table 1　Initial parameter of the simulation

Parameter	Value	Parameter	Value
R_0/m	5 000	θ_M/(°)	30
V_M/(m·s^{-1})	700	θ_T/(°)	45
V_T/(m·s^{-1})	300	b	1
a_M^{max}/g	20	a	10^5
a_T^{max}/g	12	g/(m·s^{-2})	9.8

Assume that both vehicles have the same order response dynamics (ε=1). μ_c can be calculated by Eq. (33). The covariance of available measurement noise is G=400 m^2.

Simulation results are presented in Figs. 5-10. The new adaptive weighted differential game guidance law (DGL/A) has a better performance than the differential game guidance laws with perfect information (DGL/P) and imperfect information (DGL/I). Figure 5 shows the trajectories of the missile and the target under the three differential game guidance laws. Compared with the other two, DGL/A has a smoother trajectory, and its overload does not exceed the admissible level of the interceptor control in Fig. 6.

At the beginning, the target is not maneuvering, resulting in a considerable large penalty coefficient c (Fig. 7 and Fig. 9). Once the target starts to maneuver, the penalty coefficient c decreases. The estimated error is minimal at the start, and the weight W_1 is higher than W_2. When estimated error is large, W_2 is higher than W_1.

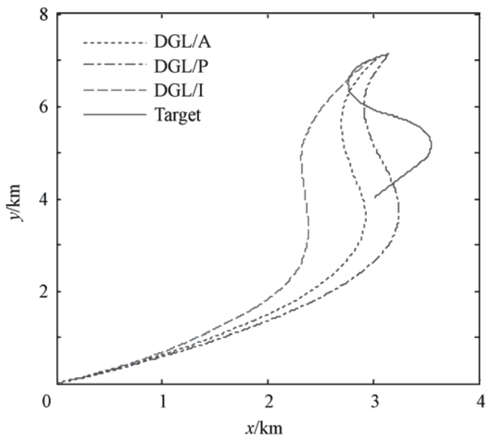

Fig. 5　Engagement plot of missile and target.

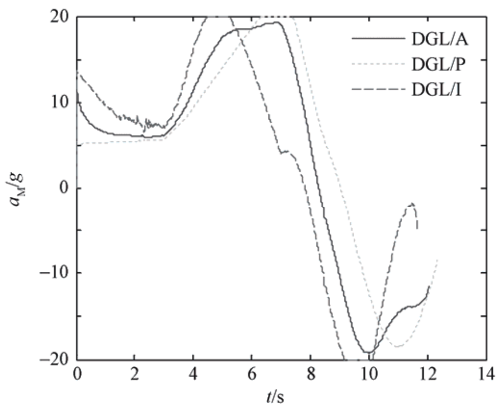

Fig. 6　Performance comparison of overload.

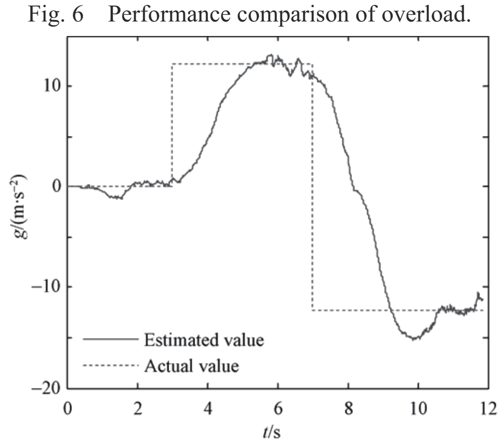

Fig. 7　Acceleration estimation.

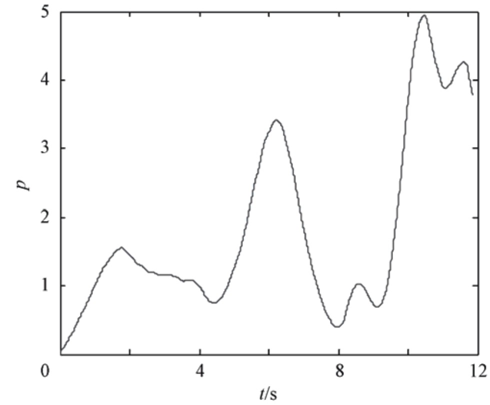

Fig. 8 Estimated error of acceleration in this simulation.

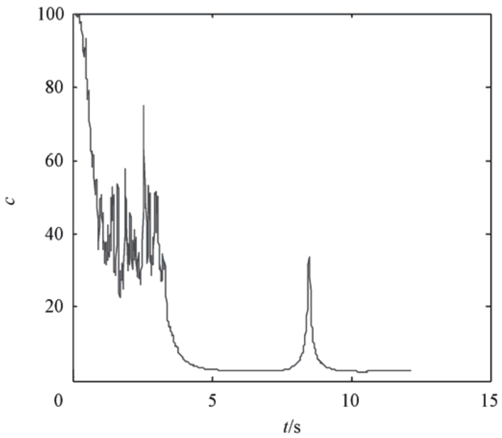

Fig. 9 Performance of penalty coefficient c.

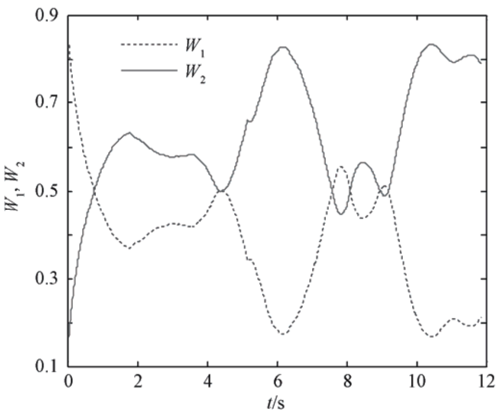

Fig. 10 Performance of adaptive weights.

Three guidance laws implemented with the estimator of Section 4 were tested using 100 Monte Carlo simulation which runs with independent random observation noise samples. In normal circumstances, the maximum warhead lethal radius of air-to-air missile is 9 m. Comparison of the statistical results of the miss distance is shown in Table 2. It is thus apparent that DGL/I ends up with unacceptable large miss distance, for using the estimation directly. In the contrast, the new guidance law uses the time varying information of target acceleration, and takes advantage of the precision of the information by adaptively combining the two guidance laws derived from different information patterns. The miss distance distribution is therefore superior.

Table 2 Performance comparison of miss distance

Guidance law	Average miss distance/m	Maximum miss distance/m
DGL/A	6.65	10.08
DGL/P	8.24	12.87
DGL/I	9.16	16.43

7. Conclusions

1) We proposed a new adaptive weighted differential game guidance law based on different information patterns. In this new law the weight factors vary according to the estimated error of target's acceleration, and its effectiveness is verified by simulation when the estimated error is large.

2) The new guidance law was tested in a scenario of intercepting a cruising UAV, which showed that the overload with this new guidance law does not exceed the admissible level of the interceptor control. This new law has a better homing performance and it is effective for intercepting modern aerial targets.

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